2501/303 2508/303
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ENGINEERING MATHEMATICS III
Oct./Nov. 2021
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN MECHANICAL ENGINEERING (PRODUCTION OPTION) (PLANT OPTION) DIPLOMA IN AUTOMOTIVE ENGINEERING DIPLOMA IN WELDING AND FABRICATION DIPLOMA IN CONSTRUCTION PLANT ENGINEERING

MODULE III

ENGINEERING MATHEMATICS III

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/Non-programmable scientific calculator.
*Answer **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as shown.
Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Determine the particular solution of the differential equation:

$x \sin y \, dx + (x^2 + 1) \cos y \, dy = 0$ given that when $x = 1, y = \frac{\pi}{2}$. (5 marks)

(b) The motion of a damped system is described by the differential equation:

$$\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = e^{3t}$$

Using the method of undetermined coefficients, solve the differential equation given that when $t = 0, y = 1$ and $\frac{dy}{dt} = 2$. (15 marks)

2. (a) Derive from first principles, the Laplace transform of $f(t) = t \cos 4t$. (9 marks)

(b) A mechanical system is described by the differential equation:

$$\frac{d^2x}{dt^2} + 9x = \sin 2t.$$

Use Laplace transforms to determine the function $x(t)$ given that $x(0) = 0$ and $x'(0) = 1$. (11 marks)

3. (a) Sketch the region of the integration and evaluate the double integral.

$$\int_0^2 \int_0^{\sqrt{(2-x^2)}} \frac{x}{\sqrt{(x^2+y^2)}} \, dy dx.$$ (6 marks)

(b) Evaluate the triple integral $\iiint_s x^2 y^2 z \, dz dy dx$, where s is defined by

$0 \le z \le (x^2 - y^2), 0 \le x \le 1, 0 \le y \le 1$. (7 marks)

(c) Use double integration to find the area bounded by the parabola $y = x^2$ and the line $y = 2x + 3$. (7 marks)

4. (a) A periodic function $f(t)$ is defined by:

$$f(t) = \begin{cases} 0, & -\pi \le t \le \frac{-\pi}{4} \\ 1, & -\frac{\pi}{4} \le t \le \frac{\pi}{4} \\ 0, & \frac{\pi}{4} \le t \le \pi \\ f(t + 2\pi), & \end{cases};$$

(i) sketch the function $f(t)$ for $-\pi \le t \le \pi$;
(ii) determine the Fourier series of $f(t)$;
(iii) by setting $t = \frac{\pi}{4}$ in the series, show that $\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}\right) = \frac{\pi}{4}$.

(14 marks)

(b) Determine the Fourier sine series of $g(t)=g(t+2)=t$ in the interval $0<t<1$.
(6 marks)

5. (a) Given that $A=\begin{bmatrix}0 & 1 & 3\\ 2 & 2 & 1\\ 0 & 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}2 & 2 & 1\\ 2 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$.

Determine:

(i) $D=A^TB$

(ii) $C=A+B$

(iii) C^{-1}
(12 mark)

(b) Three forces of a framework F_1, F_2 and F_3 are related by a system of linear equations:

$$2F_1+4F_2-6F_3=-4$$
$$F_1+5F_2+3F_3=10$$
$$F_1+3F_2+2F_3=5$$

Using Cramer's rule, determine the forces.
(8 marks)

6. (a) A scalar function $\phi(x,y,z)$ is defined by $\phi(x,y,z)=\ln(x^2+y^2+z^2)$. Determine grad ϕ at $(1,1,1)$.
(5 marks)

(b) Determine:

(i) Curl (Curl E) if $E=x^2yi-y^2zj+z^2yk$ at (2,1,1);

(ii) div V if $V=3xz^2i-2x^2yzj+2yz^4k$ at $(1,2,1)$.
(15 marks)

7. (a) Given that x_n is an approximation to the root of $f(x)=x^2-8x-5$.

(i) Using Newton-Raphson method, show that a better approximation to the root is given by:

$$x_{n+1}=\frac{2x_n^2+5}{3x_n^2-8} \text{ for } n=0,1,2,.....$$

(ii) Given that $x_0=3$, determine the value of x correct to three decimal places.
(8 marks)

(b) The data in Table I shows values obtained from an experiment.

Table I

x	1	2	3	4	5
y=f(x)	2	5	10	17	26

Using Newton-Gregory interpolation formula, determine:

(i) $f(1.5)$;

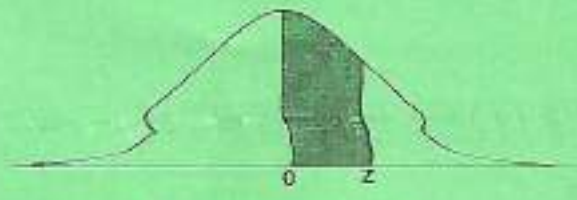
(ii) $f(4.5)$. (12 marks)

8. (a) The equation of a circle is given by $x^2 + y^2 - 4x + 2y - 4 = 0$. Determine its centre and radius, then sketch the circle. (4 marks)

(b) The equation of a circle passes through the points (0, 1), (1, 2) and (3, 1). Determine the locus of these points, then sketch the locus. (8 marks)

(c) Determine the angle between

$\underline{A} = 2\underline{i} + 2\underline{j} - \underline{k}$ and $\underline{B} = 6\underline{i} - 3\underline{j} + 2\underline{k}$. (8 marks)



Normal Probability

	Area under the standard normal curve from 0 to Z									
Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.000000	0.003989	0.007978	0.011966	0.015953	0.019939	0.023922	0.027903	0.031881	0.035856
0.1	0.039828	0.043795	0.047758	0.051717	0.055670	0.059618	0.063559	0.067495	0.071424	0.075345
0.2	0.079260	0.083166	0.087064	0.090954	0.094835	0.098706	0.102568	0.106420	0.110261	0.114092
0.3	0.117911	0.121720	0.125516	0.129300	0.133072	0.136831	0.140576	0.144309	0.148027	0.151732
0.4	0.155422	0.159097	0.162757	0.166402	0.170031	0.173645	0.177242	0.180822	0.184386	0.187933
0.5	0.191462	0.194974	0.198468	0.201944	0.205401	0.208840	0.212260	0.215661	0.219043	0.222405
0.6	0.225747	0.229069	0.232371	0.235653	0.238914	0.242154	0.245373	0.248571	0.251748	0.254903
0.7	0.258036	0.261148	0.264238	0.267305	0.270350	0.273373	0.276373	0.279350	0.282305	0.285236
0.8	0.288145	0.291030	0.293892	0.296731	0.299546	0.302337	0.305105	0.307850	0.310570	0.313267
0.9	0.315940	0.318589	0.321214	0.323814	0.326391	0.328944	0.331472	0.333977	0.336457	0.338913
1.0	0.341345	0.343752	0.346136	0.348495	0.350830	0.353141	0.355428	0.357690	0.359929	0.362143
1.1	0.364334	0.366500	0.368643	0.370762	0.372857	0.374928	0.376976	0.379000	0.381000	0.382977
1.2	0.384930	0.386861	0.388768	0.390651	0.392512	0.394350	0.396165	0.397958	0.399727	0.401475
1.3	0.403200	0.404902	0.406582	0.408241	0.409877	0.411492	0.413085	0.414657	0.416207	0.417736
1.4	0.419243	0.420730	0.422196	0.423641	0.425066	0.426471	0.427855	0.429219	0.430563	0.431888
1.5	0.433193	0.434478	0.435745	0.436992	0.438220	0.439429	0.440620	0.441792	0.442947	0.444083
1.6	0.445201	0.446301	0.447384	0.448449	0.449497	0.450529	0.451543	0.452540	0.453521	0.454486
1.7	0.455435	0.456367	0.457284	0.458185	0.459070	0.459941	0.460796	0.461636	0.462462	0.463273
1.8	0.464070	0.464852	0.465620	0.466375	0.467116	0.467843	0.468557	0.469258	0.469946	0.470621
1.9	0.471283	0.471933	0.472571	0.473197	0.473810	0.474412	0.475002	0.475581	0.476148	0.476705
2.0	0.477250	0.477784	0.478308	0.478822	0.479325	0.479818	0.480301	0.480774	0.481237	0.481691
2.1	0.482136	0.482571	0.482997	0.483414	0.483823	0.484222	0.484614	0.484997	0.485371	0.485738
2.2	0.486097	0.486447	0.486791	0.487126	0.487455	0.487776	0.488089	0.488396	0.488696	0.488989
2.3	0.489276	0.489556	0.489830	0.490097	0.490358	0.490613	0.490863	0.491106	0.491344	0.491576
2.4	0.491802	0.492024	0.492240	0.492451	0.492656	0.492857	0.493053	0.493244	0.493431	0.493613
2.5	0.493790	0.493963	0.494132	0.494297	0.494457	0.494614	0.494766	0.494915	0.495060	0.495201
2.6	0.495339	0.495473	0.495604	0.495731	0.495855	0.495975	0.496093	0.496207	0.496319	0.496427
2.7	0.496533	0.496636	0.496736	0.496833	0.496928	0.497020	0.497110	0.497197	0.497282	0.497365
2.8	0.497445	0.497523	0.497599	0.497673	0.497744	0.497814	0.497882	0.497948	0.498012	0.498074
2.9	0.498134	0.498193	0.498250	0.498305	0.498359	0.498411	0.498462	0.498511	0.498559	0.498605
3.0	0.498650	0.498694	0.498736	0.498777	0.498817	0.498856	0.498893	0.498930	0.498965	0.498999
3.1	0.499032	0.499065	0.499096	0.499126	0.499155	0.499184	0.499211	0.499238	0.499264	0.499289
3.2	0.499313	0.499336	0.499359	0.499381	0.499402	0.499423	0.499443	0.499462	0.499481	0.499499
3.3	0.499517	0.499534	0.499550	0.499566	0.499581	0.499596	0.499610	0.499624	0.499638	0.499651
3.4	0.499663	0.499675	0.499687	0.499698	0.499709	0.499720	0.499730	0.499740	0.499749	0.499758
3.5	0.499767	0.499776	0.499784	0.499792	0.499800	0.499807	0.499815	0.499822	0.499828	0.499835
3.6	0.499841	0.499847	0.499853	0.499858	0.499864	0.499869	0.499874	0.499879	0.499883	0.499888
3.7	0.499892	0.499896	0.499900	0.499904	0.499908	0.499912	0.499915	0.499918	0.499922	0.499925
3.8	0.499928	0.499931	0.499933	0.499936	0.499938	0.499941	0.499943	0.499946	0.499948	0.499950
3.9	0.499952	0.499954	0.499956	0.499958	0.499959	0.499961	0.499963	0.499964	0.499966	0.499967
4.0	0.499968	0.499970	0.499971	0.499972	0.499973	0.499974	0.499975	0.499976	0.499977	0.499978
4.1	0.499979	0.499980	0.499981	0.499982	0.499983	0.499983	0.499984	0.499985	0.499985	0.499986
4.2	0.499987	0.499987	0.499988	0.499988	0.499989	0.499989	0.499990	0.499990	0.499991	0.499991
4.3	0.499991	0.499992	0.499992	0.499993	0.499993	0.499993	0.499993	0.499994	0.499994	0.499994
4.4	0.499995	0.499995	0.499995	0.499995	0.499996	0.499996	0.499996	0.499996	0.499996	0.499996
4.5	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499997	0.499998	0.499998	0.499998
4.6	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499998	0.499999	0.499999
4.7	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.8	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999	0.499999
4.9	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000
5.0	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000	0.500000

TABLE OF LAPLACE TRANSFORM FORMULAS

$$\mathcal{L}[t^n] = \frac{n!}{s^{n+1}} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^n}\right] = \frac{1}{(n-1)!}t^{n-1}$$

$$\mathcal{L}[e^{at}] = \frac{1}{s-a} \qquad \mathcal{L}^{-1}\left[\frac{1}{s-a}\right] = e^{at}$$

$$\mathcal{L}[\sin at] = \frac{a}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{1}{s^2+a^2}\right] = \frac{1}{a}\sin at$$

$$\mathcal{L}[\cos at] = \frac{s}{s^2+a^2} \qquad \mathcal{L}^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

First Differentiation Formula

$$\mathcal{L}[f^{(n)}(t)] = s^n\,\mathcal{L}[f(t)] - s^{n-1}f(0) - s^{n-2}f'(0) - \ldots - f^{(n-1)}(0)$$

$$\mathcal{L}\left[\int_0^t f(u)\,du\right] = \frac{1}{s}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{1}{s}F(s)\right] = \int_0^t \mathcal{L}^{-1}[F(s)]\,du$$

In the following formulas, $F(s) = \mathcal{L}[f(t)]$ so $f(t) = \mathcal{L}^{-1}[F(s)]$.

First Shift Formula

$$\mathcal{L}[e^{at}f(t)] = F(s-a) \qquad \mathcal{L}^{-1}[F(s)] = e^{at}\,\mathcal{L}^{-1}[F(s+a)]$$

Second Differentiation Formula

$$\mathcal{L}[t^n f(t)] = (-1)^n\frac{d^n}{ds^n}\mathcal{L}[f(t)] \qquad \mathcal{L}^{-1}\left[\frac{d^nF(s)}{ds^n}\right] = (-1)^n t^n f(t)$$

Second Shift Formula

$$\mathcal{L}[u_a(t)g(t)] = e^{-as}\,\mathcal{L}[g(t+a)] \qquad \mathcal{L}^{-1}[e^{-as}F(s)] = u_a(t)f(t-a)$$

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